Phenomenology of the Λ/Σ^0 production ratio in pp collisions

A. Sibirtsev[1], J. Haidenbauer[2,a], H.-W. Hammer[1], and U.-G. Meißner[1,2]

[1] Helmholtz-Institut für Strahlen- und Kernphysik (Theorie), Universität Bonn, Nußallee 14-16, D-53115 Bonn, Germany
[2] Institut für Kernphysik (Theorie), Forschungszentrum Jülich, D-52425 Jülich, Germany

Abstract. We show that the recently measured asymmetry in helicity-angle spectra of the Λ-hyperons, produced in the reaction $pp \to K^+ \Lambda p$ reaction, and the energy dependence of the total $pp \to K^+ \Lambda p$ cross-section can be explained consistently by the same Λp final-state interaction. Assuming that there is no final-state interaction in the $\Sigma^0 p$ channel, as suggested by the available data for the reaction $pp \to K^+ \Sigma^0 p$, we can also reproduce the energy dependence of the Λ/Σ^0 production ratio and, in particular, the rather large ratio observed near the reaction thresholds. The nominal ratio of the Λ and Σ^0 production amplitudes squared, *i.e.* when disregarding the final-state interaction, turns out to be about 3, which is in line with hyperon production data from proton and nuclear targets available at high energies.

PACS. 13.75.Ev Hyperon-nucleon interactions – 14.20.Jn Hyperons – 25.40.Ve Other reactions above meson production thresholds (energies > 400 MeV)

One of the surprising results observed at the COSY accelerator facility is the large ratio of the $pp \to K^+ \Lambda p$ to $pp \to K^+ \Sigma^0 p$ cross-sections near the reaction thresholds [1,2]. This ratio is as large as 28 [1] at very low energies and eventually approaches values around 3 with increasing energy. In the last few years several theoretical studies [3–8] appeared where different production scenarios were considered in order to describe simultaneously both the $pp \to K^+ \Lambda p$ and $pp \to K^+ \Sigma^0 p$ reaction cross-sections and, thus, the Λ/Σ^0 ratio. These included π, K as well as heavier meson exchanges, intermediate baryonic resonances coupled to the $K^+ \Lambda$ and $K^+ \Sigma^0$ channels but also the Λp and $\Sigma^0 p$ final-state interaction (FSI). However, as summarized in refs. [2,9], none of these models is able to reproduce the energy dependence of the Λ/Σ^0 ratio convincingly.

A basic problem is that the relevance of the FSI on one hand side, and the interplay between the different baryonic resonances on the other side cannot be resolved from considering only the total reaction cross-section. Therefore, in the present paper we follow a different strategy. We look not only at the total cross-sections but also at differential observables because some of them do allow to distinguish between effects from the FSI and baryonic resonances, as we pointed out recently [10]. Moreover, for the reaction $pp \to K^+ \Lambda p$ data for the relevant observables are already available in the literature [11]. Note that our study is completely phenomenological in the sense that we do not con-

sider any specific reaction mechanism. But we take into account the effects from the Λp final-state interaction. We will argue that the differential data of ref. [11] strongly support the interpretation of the measured energy dependence of the total $pp \to K^+ \Lambda p$ cross-section in terms of FSI effects. At the same time, and in view of the lack of any visible FSI effects in the reaction $pp \to K^+ \Sigma^0 p$ [2], we come to the conclusion that the energy dependence of the Λ/Σ^0 cross-section ratio could likewise be governed primarily by FSI effects in the $pp \to K^+ \Lambda p$ channel.

The role of the Λp FSI in describing the $pp \to K^+ \Lambda p$ cross-section near the reaction threshold was already addressed after the first experiments of the COSY-11 Collaboration [12,13] for excess energies below 7 MeV. But only further measurements by the COSY-11 and COSY-TOF Collaborations at higher energies explicitly indicated [1, 2,14] that the energy dependence of the $pp \to K^+ \Lambda p$ cross-section deviates from the phase space, *i.e.* the expected ϵ^2 behavior. (The excess energy ϵ is defined as $\epsilon = \sqrt{s} - m_K - m_\Lambda - m_p$, where s is the squared invariant collision energy, while m_K, m_Λ and m_p are the masses of the kaon, the Λ-hyperon and the proton, respectively.) This deviation from the phase-space behavior —in particular an enhancement at lower collision energies— was interpreted as an effect due to the Λp FSI in several investigations [3,4,6,15,16].

However, the near-threshold enhancement could also result from the contributions of resonances coupled to the meson-baryon ($K^+ \Lambda$) subsystem. Indeed, varying the

ᵃ e-mail: `j.haidenbauer@fz-juelich.de`

mass and width of such resonances one can reproduce the energy dependence close to the reaction threshold too. For example, assuming that the $N^*(1535)$ resonance couples to the $K\Lambda$ system as strongly as to ηN the authors of ref. [17] came to the conclusion that the near-threshold enhancement observed in the $pp \to K^+\Lambda p$ cross-section data is caused primarily by the $N^*(1535)$ contribution. In their calculation the Λp FSI was entirely neglected!

Thus, it is obvious that the low-energy enhancement of the $pp \to K^+\Lambda p$ cross-section can be described by the Λp FSI as well as by the contribution of resonances coupled to the $K^+\Lambda$ system with the resonance mass below $m_K + m_\Lambda$. As already mentioned, to resolve this issue one needs to analyze differential data. For example, it is obvious that a low-mass enhancement of the Λp invariant mass spectrum would reflect the importance of the FSI. Corresponding data on the Λp mass spectra were already available for some time, though only for $\epsilon > 250\,\text{MeV}$. Invariant mass spectra can be obtained from the K^+-meson momentum spectra measured at certain angles in pp collisions [18–20]. Although the K^+-meson spectra were measured for the inclusive reaction, i.e. $pp \to K^+ X$, the contribution from the exclusive $pp \to K^+\Lambda p$ channel can be well isolated by analyzing the missing mass spectra below the Σ-hyperon production threshold. The data provided in refs. [19,20] at different proton beam energies and K^+-meson production angles indicate a substantial enhancement at low Λp invariant masses with respect to the pure phase-space distribution. Actually, the data from ref. [20] were even used in attempts to determine the Λp effective-range parameters [21,22]. Therefore, there is no doubt that the Λp FSI plays a substantial role in the reaction $pp \to K^+\Lambda p$.

On the other hand, it is still an open question how strongly the FSI contributes at COSY energies and, in particular, whether the Λp FSI is entirely responsible for the low-energy enhancement. To answer this question one requires differential observables for the $pp \to K^+\Lambda p$ reaction at COSY energies. Such data were recently provided by the COSY-TOF Collaboration who measured [11] the angular distributions of the Λ-hyperon in reference to the proton direction in the $K^+\Lambda$ center-of-mass system for the $pp \to K^+\Lambda p$ reaction at the beam momentum of $2.85\,\text{MeV}/c$ ($\epsilon = 171\,\text{MeV}$). The angular spectra are almost isotropic for low invariant masses of the $K^+\Lambda$ system, i.e. for $M_{K\Lambda} < 1.69\,\text{GeV}$, cf. fig. 1. However, at larger $K^+\Lambda$ masses the measured angular spectra exhibit an anisotropy, which becomes more substantial with increasing $M_{K\Lambda}$. Note that the maximal $M_{K\Lambda}$ corresponds to the minimal invariant mass $M_{\Lambda p}$ of the Λp system.

The measured angular spectra, which are related to the so-called helicity-angle (θ_H) spectra, can be expressed [10] in terms of the invariant masses of the $K^+\Lambda$ and Λp subsystems by expanding the Λp invariant mass. The helicity angle of the Λ-hyperon is then given as

$$\cos\theta_H = \left[2M_{K\Lambda}^2(m_p^2 + m_\Lambda^2 - M_{\Lambda p}^2) + (s - M_{K\Lambda}^2 - m_p^2)\right.$$
$$\times(M_{K\Lambda}^2 + m_\Lambda^2 - m_K^2)\left]\lambda^{-1/2}(s, M_{K\Lambda}^2, m_p^2)\right.$$
$$\times\lambda^{-1/2}(M_{K\Lambda}^2, m_\Lambda^2, m_K^2), \qquad (1)$$
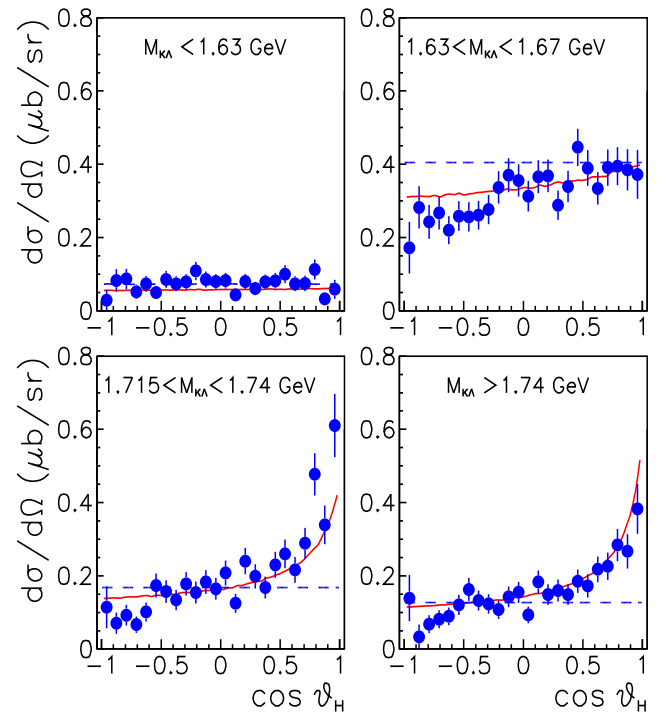

Fig. 1. The helicity-angle distribution of the Λ-hyperon for different $K^+\Lambda$ invariant masses. The data are from ref. [11], obtained at the excess energy $\epsilon = 171\,\text{MeV}$. The solid lines show our calculations including the Λp FSI, while the dashed lines are results obtained by switching off the FSI.

where $\lambda(x,y,z) = (x-y-z)^2 - 4yz$. For fixed $M_{K\Lambda}$ the angle $\cos\theta_H = 1$ corresponds to the minimal invariant mass of the Λp subsystem. Therefore, the Λp FSI should manifest itself as an enhancement at forward θ_H angles[1]. Equation (1) shows that by applying different cuts on $M_{K\Lambda}$ one can study the spectra at different $M_{\Lambda p}$ ranges and, at sufficiently large s, one can isolate the Λp FSI.

In the following study we explore the effects of the Λp FSI. It is included in our calculation within the Watson-Migdal approach [23] by factorizing the reaction amplitude in terms of a practically constant production amplitude $\mathcal{M}_0$ and an FSI factor, i.e.

$$\mathcal{M} \approx \mathcal{M}_0 \times \mathcal{A}_{\Lambda p}, \qquad (2)$$

where the latter is taken to be the Jost function,

$$\mathcal{A}_{\Lambda p}(q) = \frac{q + i\beta}{q - i\alpha}, \qquad (3)$$

The parameters α and β are related to the Λp effective-range parameters [10]. In the present work we employ the same values ($\alpha = -72.3\,\text{MeV}/c$, $\beta = 212.7\,\text{MeV}/c$

---

[1] In our convention the sign of θ_H is opposite to the one in the experimental analysis [11]. In the following we use our sign convention since it follows directly from the Dalitz plot representation. Therefore, the experimental angular distributions were transformed to helicity θ_H-angle spectra by changing the sign of the experimental angle.
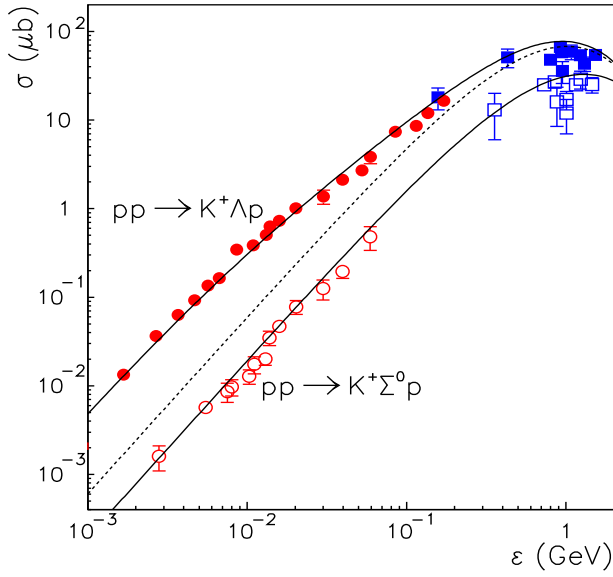
Fig. 2. Total cross-sections for the $pp \to K^+ \Lambda p$ (closed symbols) and $pp \to K^+ \Sigma^0 p$ (open symbols) reactions as a function of the excess energy ϵ. Results from COSY [1,2,11,13,14] are indicated by circles, while the squares are data from ref. [24]. The solid lines are our results for the Λ and Σ^0 reaction channels, respectively. The dashed line is obtained by switching off the Λp final-state interaction.

corresponding to the Λp effective-range parameters $a = -1.8\,\text{fm}$, $r = 2.8\,\text{fm}$) that we already used in ref. [10]. With those the energy dependence of the $pp \to K^+ \Lambda p$ reaction cross-section could be described over a large energy range. For the production amplitude $\mathcal{M}_0$, however, we use here a simple parametrization (see below) so that the total cross-section can be evaluated analytically [10]:

$$\sigma(\epsilon) = \frac{\sqrt{m_K m_N m_\Lambda}}{2^7 \pi^2 (m_K + m_N + m_\Lambda)^{3/2}} \frac{\epsilon^2}{\sqrt{s^2 - 4sm_N^2}} |\mathcal{M}_0|^2$$

$$\times \left[1 + \frac{4\beta^2 - 4\alpha^2}{(-\alpha + \sqrt{\alpha^2 + 2\mu\epsilon})^2} \right]. \quad (4)$$

Here μ is the reduced mass of the hyperon-nucleon system. Note that the last term in the square brackets of eq. (4) arises from taking into account the Λp FSI.

With regard to $\mathcal{M}_0$ we allow for a smooth (exponential) dependence on the excess energy in order to be able to connect with the data at higher energies. To be concrete we use $|\mathcal{M}_0|^2 = 1.89 \exp(-1.3\epsilon)$ μb (ϵ in GeV) which fits the $pp \to K^+ \Lambda p$ cross-section data [1,2,11, 13,14,24] rather well as is shown in fig. 2 by the solid line. At the energy $\epsilon = 171\,\text{MeV}$, where the helicity-angle distributions were measured, we obtain $18.5\,\mu$b which is in reasonable agreement with the experimental value of $16.5 \pm 0.4\,\mu$b [11]. We would like to emphasize that the energy dependence introduced by $\mathcal{M}_0$ is rather small over the range covered by the COSY data (solid circles in fig. 2). It amounts to a variation of only about 20%. This has to be compared with the effect of the Λp FSI which

produces an enhancement of a factor 8 [10], in the same energy range, cf. the solid and dashed curves in fig. 2.

Predictions for the helicity-angle distribution of the Λ-hyperon are presented in fig. 1 together with the experimental angular spectra. Since the data are given in arbitrary units [11] we have normalized them to our calculations. But we use the same overall normalization for all the different $M_{K\Lambda}$ cuts. Also the results without the Λp FSI (dashed lines in fig. 1) are based on the same normalization.

Obviously, the experimental result of ref. [11] can be naturally explained in terms of the Λp FSI. The calculation without this FSI fails completely. Thus, we consider these angular spectra as a strong direct evidence for the presence of the Λp FSI. Most intriguing, however, is the fact that the specific strength of the Λp FSI we employed reproduces the measured helicity-angle distributions rather nicely and at the same time it also yields an excellent description of the total $pp \to K^+ \Lambda p$ cross-section. This strongly suggests that the enhancement in the latter quantity at lower collision energies is primarily due to the Λp FSI.

Let us now come to the reaction $pp \to K^+ \Sigma^0 p$. Corresponding cross-section data are shown by open symbols in fig. 2. Again, the circles are results of experiments at COSY [1,2], while the squares are from ref. [24]. The solid line shows the calculation by eq. (4) (replacing only m_Λ by m_Σ and considering that ϵ is now defined by $\epsilon = \sqrt{s} - m_K - m_\Sigma - m_p$) and without any $\Sigma^0 p$ FSI, *i.e.* omitting the last term in the square brackets of eq. (4). For the production amplitude $\mathcal{M}_0$ we use the same form as before but with a readjusted normalization: $|\mathcal{M}_0|^2 = 0.61 \exp(-1.3\epsilon)$ μb. We kept the smooth exponential energy dependence the same as before because a) the $pp \to K^+ \Sigma^0 p$ reaction cross-sections available for $\epsilon > 300\,\text{MeV}$ have large uncertainties so that a determation of this parameter from a fit to those data is not possible anyway, and b) it is convenient to have the same energy dependence as for the reaction $pp \to K^+ \Lambda p$ because then the Λ/Σ^0 cross-section ratio approaches a constant at high energies.

Obviously, the calculation without $\Sigma^0 p$ FSI yields already a perfect reproduction of the energy dependence of the $pp \to K^+ \Sigma^0 p$ data over the whole considered energy range. Especially, in the COSY regime the data are completely in line with the pure phase-space (ϵ^2) dependence. Based on this evidence one would conclude that effects from the $\Sigma^0 p$ FSI are much smaller than those in the Λp channel, as already pointed out in ref. [2]. Clearly, like for $pp \to K^+ \Lambda p$ discussed above, one should keep in mind that firm conclusions can only be drawn once one has also inspected the helicity-angle spectra for the Σ^0. Corresponding data at excess energies $60 < \epsilon < 210\,\text{MeV}$ have already been taken by the TOF Collaboration and are presently analyzed [25]. It will be very interesting to see whether those data are in line with the absence of any $\Sigma^0 p$ FSI as conjectured from the total cross-section. In such a case one expects that the $\cos\theta_H$ distribution is completely isotropic for large $K^+ \Sigma^0$ invariant masses.
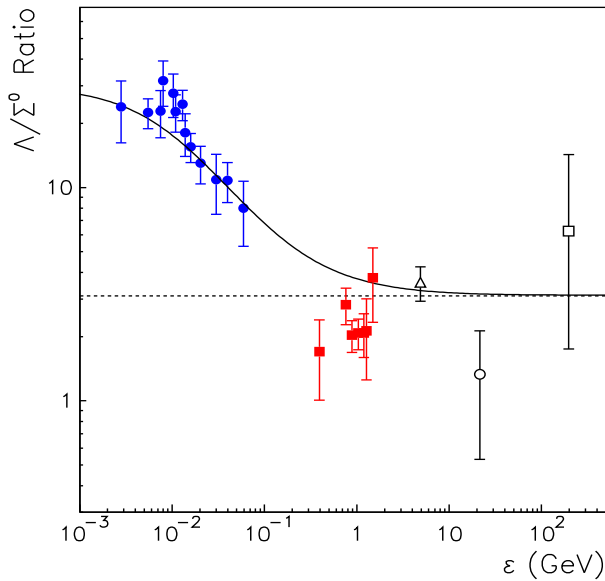
Fig. 3. The Λ/Σ^0 cross-section ratio as a function of the excess energy ϵ. The solid circles show the ratio obtained for the $pp \to K^+\Lambda p$ and $pp \to K^+\Sigma^0 p$ reactions at COSY [2]. Solid squares are pp results from ref. [24]. The open triangle and open circle are ratios measured in p Be [28] and p Ne [29] collisions, respectively. The open square is the result from a d Au experiment [30]. The curves are cross-section ratios based on the $pp \to K^+\Lambda p$ results with Λp FSI (solid line) and without FSI (dashed line).

Anyway, the lack of apparent FSI effects in the $\Sigma^0 p$ channel is certainly quite suprising. But it could be due to delicate cancellations between effects resulting from the two possible transitions in the final state, namely $\Sigma^0 p \to \Sigma^0 p$ and $\Sigma^+ n \to \Sigma^0 p$.

Based on our parametrizations of the production amplitudes $\mathcal{M}_0$ we can now calculate the ratio $\sigma_{(pp \to K^+\Lambda p)}/\sigma_{(pp \to K^+\Sigma^0 p)}$. The ratio of the squared production amplitudes alone yields 3.1, in rough agreement with the experimental ratio at higher energies. Since the enhancement in the $pp \to K^+\Lambda p$ cross-section at lower energies was found to be primarily due to the Λp FSI, it follows immediately that also the near-threshold enhancement of the Λ/Σ^0 ratio must stem entirely from the Λp FSI. Our results are illustrated in fig. 3. The solid line is obtained with inclusion of the Λp FSI in eq. (4) utilizing those FSI parameters (α, β) that describe consistently the helicity-angle distribution of the Λ-hyperon [11]. The dashed line in fig. 3 shows the ratio of the squared production amplitude for the $pp \to K^+\Lambda p$ and $pp \to K^+\Sigma^0 p$ reactions. They are compared with measurements from COSY [2] (solid circles) and with data at higher energies [24] (solid squares).

It is worthwhile to mention that the Λ/Σ^0 ratio has also been discussed in the context of reactions with nuclear targets. Quark model calculations predict [26,27] that in inclusive reactions at high energies the Λ-multiplicity has to be much larger —about 8 times— than that of the Σ-hyperon. This does not violate $SU(3)$ symmetry but re-

sults from the decay of heavy baryonic resonances, which effectively enhances the Λ production rate. Figure 3 contains the Λ/Σ^0 ratio observed in p Be (open triangle) [28] and p Ne (open circle) [29] collisions. The open square is a very recent result [30] from the STAR Collaboration at RHIC obtained for d Au collisions at $\sqrt{s} = 200$ GeV.

It is interesting to observe that the ratios for nuclear targets, measured at high energies, are roughly in line with the results from high-energy pp collisions. Unfortunately, the new and still preliminary STAR result is afflicted by large uncertainties and, thus, precludes any firm conclusion concerning a possibly larger ratio with respect to that found in the pp interactions. Several authors have pointed out that the experimental ratio of around 3 coincides with the ratio of the isospin multiplicity of the Λ and Σ's [2, 28,30]. But we are not aware of any deeper reason why those two quantities should be connected.

In conclusion we analyzed the helicity-angle spectra of Λ hyperons produced in the reaction $pp \to K^+\Lambda p$, measured recently by the COSY-TOF Collaboration. We argued that the observed anisotropy of the angular distribution at large invariant masses of the $K^+\Lambda$ system is a strong evidence for the presence of the Λp FSI. Adopting a specific strength of the Λp FSI the measured helicity-angle distributions could be reproduced quantitatively (within a Watson-Migdal approach). The same Λp FSI yields also an excellent description of the total $pp \to K^+\Lambda p$ cross-section which strongly suggests that the enhancement in the latter quantity at lower collision energies is indeed primarily due to the Λp FSI. Thus, our result casts considerable doubts on a recent interpretation of this enhancement in terms of a large contribution from the $N^*(1535)$ resonance [17] (see also ref. [31]). In view of the lack of any visible FSI effects in the reaction $pp \to K^+\Sigma^0 p$ this implies that the energy dependence of the Λ/Σ^0 ratio could then be likewise driven by the Λp FSI. However, for the final conclusion on this issue data on the helicity-angle distribution of the Σ^0-hyperon are necessary. The nominal ratio of the Λ and Σ^0 production amplitudes squared, *i.e.* disregarding FSI effects, amounts to 3.1, which is in line with available data from proton and nuclear targets at high energies.

References

1. S. Sewerin *et al.*, Phys. Rev. Lett. **83**, 682 (1999) [nucl-ex/9811004].

2. P. Kowina *et al.*, Eur. Phys. J. A **22**, 293 (2004) [nucl-ex/0402008].

3. A. Gasparian, J. Haidenbauer, C. Hanhart, L. Kondratyk, J. Speth, Phys. Lett. B **480**, 273 (2000) [nucl-th/9909017].

4. A. Sibirtsev, K. Tsushima, W. Cassing, A.W. Thomas, nucl-th/0004022.
5. L.-M. Laget, Nucl. Phys. A **691**, 11c (2001).
6. R. Shyam, G. Penner, U. Mosel, Phys. Rev. C **63**, 022202 (2001) [nucl-th/0010102].
7. R. Shyam, Phys. Rev. C **73**, 035211 (2006) [nucl-th/0512007].
8. M. Dillig, M. Schott, nucl-th/0604059.
9. T. Rożek *et al.*, to be published in Phys. Lett. B [nucl-ex/0607034].
10. A. Sibirtsev, J. Haidenbauer, H.-W. Hammer, S. Krewald, Eur. Phys. J. A **27**, 269 (2006) [nucl-th/0512059].
11. S. Abd El-Samad *et al.*, Phys. Lett. B **632**, 27 (2006).
12. J.T. Balewski *et al.*, Phys. Lett. B **388**, 859 (1996).
13. J.T. Balewski *et al.*, Phys. Lett. B **420**, 211 (1998).
14. R. Bilger *et al.*, Phys. Lett. B **420**, 217 (1998).
15. G. Fäldt, C. Wilkin, Z. Phys. A **357**, 241 (1997) [nucl-th/9612019].
16. C. Hanhart, Phys. Rep. **397**, 155 (2004) [hep-ph/0311341].
17. B.C. Liu, B.S. Zou, Phys. Rev. Lett. **96**, 042002 (2006).
18. W.J. Hogan *et al.*, Phys. Rev. **166**, 1472 (1968).
19. J.T. Reed *et al.*, Phys. Rev. **168**, 168 (1968).
20. R. Siebert *et al.*, Nucl. Phys. A **567**, 819 (1994).
21. A. Gasparyan, J. Haidenbauer, C. Hanhart, J. Speth, Phys. Rev. C **69**, 034006 (2004) [hep-ph/0311116]; A. Gasparyan, J. Haidenbauer, C. Hanhart, Phys. Rev. C **72**, 034006 (2005) [nucl-th/0506067].
22. F. Hinterberger, A. Sibirtsev, Eur. Phys. J. A **21**, 313 (2004) [nucl-ex/0402021].
23. M.L. Goldberger, K.M. Watson, *Collision Theory* (John Wiley and Sons, 1967).
24. A. Baldini, V. Flaminio, W.G. Moorhear, D.R.O. Morrison, Landolt-Börnstein, New Ser. **12** (1988).
25. TOF-COSY Collaboration, private communication.
26. V.V. Anisovich, V.M. Shekter, Nucl. Phys. B **55**, 455 (1973).
27. J.D. Bjorken, G.R. Farrar, Phys. Rev. D **9**, 1449 (1974).
28. M.W. Sullivan *et al.*, Phys. Rev. D **36**, 674 (1987).
29. B.S. Yuldashev *et al.*, Phys. Rev. D **43**, 2792 (1991).
30. G. Van Buren *et al.*, J. Phys. G **31**, S195; S1127 (2005); Rom. Rep. Phys. **58**, 069 (2006) [nucl-ex/0512018].
31. A. Sibirtsev, J. Haidenbauer, U.-G. Meißner, hep-ph/0607212.